OPTIMUM THRESHOLDS FOR EFFECTIVE MARKET SURVEILLANCE VIA NOVEL PRICE MODELS

Asoka Korale, Ph.D.
London Stock Exchange Technology, Malabe, Sri Lanka,
asoka.j.m.korale@gmail.com

***Abstract*— Unusual movements in prices may indicate manipulative behavior or be the result of a market anomaly that could serve as a precursor to a sequence of events that could lead to major losses. Detecting these events in order to ensure investor confidence and an efficient market is the province of market surveillance. This paper presents novel methods by which to model prices and place limits on its movement ensuring that such unusual behavior is detected in time to effect preventive measures. Market surveillance is often affected by a series of software configurable alerts that depend on a set of thresholds to detect when prices are outside certain desired limits. An optimum set of such thresholds can be estimated by analyzing historical prices to estimate the likelihood of observing unusual movements leading to a more effective solution. In one contribution we estimate the random process governing the movement in the price and simulate a series of realizations drawn from the same underlying process to gain insights into unusual behavior that could manifest. In this modeling, we estimate the distribution of the sequence of random variables governing the prices and generate a series of realizations or paths with the same underlying distribution. We also develop an equation to predict the expected deviation in price between two points in a sequence of prices and a measure of the uncertainty in this deviation. In another contribution we model prices as a linear regression on consecutive prices to estimate its movement and arrive at an estimate for the distribution in the error of such a prediction. By these techniques we estimate the trend and maximum deviation in price that could be expected over a sequence of prices in order to optimize the alert thresholds. Through these analyses we also observe that the variance in the price is dependent on the number of samples in a sequence of prices over which the measurement is made due to the behavior of the random process governing its movement and propose that the variance be estimated over a fixed number of consecutive prices ensuring a more stable and consistent estimate. By these contributions we envisage a more stable trading environment with lower risks facilitated by an effective surveillance solution enabled by a set of optimum thresholds that are estimated by a more accurate characterization of the movement in the price.**

Keywords*— linear regression; random walk; volatility; market surveillance;

I. INTRODUCTION

Detecting unusual movements in traded price provides clues to manipulative behavior in the capital market. Preventing such behavior is key to ensuring investor confidence and an efficient market. Large variations in price may also manifest due to the influence of market impacting external events or news that changes the overall demand or supply for securities. The interactions between automated trading systems and the algorithms responsible for trading may also cause such unusual variations when the confluence of an unforeseen sequence of events cause these systems to behave in ways that cause wide variations in the traded price. These interactions between trading algorithms have the potential to cause massive losses across the market impacting a wide range of financial instruments.

Monitoring such variations in the traded price and the establishment of a-priori limits on its allowable movement is essential to maintain a stable operating environment and confidence in the market. The need to model prices and place apriori limits on its movement is thus established and, in this contribution,, we present several models by which to quantify its behavior in order to establish these limits.

Capital market surveillance systems traditionally employ a system of alerts that are triggered when the prices move beyond preset limits. The limits are typically governed by a set of software configurable thresholds. These thresholds need calibration in order to effectively capture situations where the price movements are genuinely anomalous. The models developed in this paper facilitate a better estimate of these unusual movements by a more accurate modeling of the random processes underlying the prices. Thereby enabling a better estimate for the alert thresholds.

The variance in the price plays a key role in many algorithms and models used to characterize prices making a reliable estimate of its characteristics of crucial importance. Through our analysis we also observe that the variance in prices is impacted by the number of consecutive prices over which the estimate is made. In light of this we propose solutions for improving the accuracy of its measurement and discuss its sensitivity to different sampling techniques.

II. COMMON MODELS OF PRICE

It is common practice to estimate a model using historical prices with the aim of simulating the prices. Once a model has been estimated, prices can be simulated using the model in order to gauge the potential variations that may manifest in a way that is consistent with the parameters of the model. A wide variety of models with varying degrees of complexity may be employed. Principal among them are the random walk, models with trend and with mean reversion. The parameters of the models are estimated from historical prices often using least squares optimization methods.

Each model so estimated can be used to simulate the price so that potential variations that may be observed are quantified. The likelihood of observing certain anomalous behaviour may be gauged from observations based on these analyses.

A. Simulating prices with constant volatility models [1]

In this type of model, the normality or log normality of the prices is commonly assumed. This thinking is in line with estimating a measure of the volatility of the price from historical data and scaling a standard normal random variable to generate another random variable with the desired variance.

Constant variance over the period of interest is usually assumed in this type of model. The volatility of the prices is commonly estimated by measuring the variance in historical prices. The change in prices is then modelled by a zero mean normal random variable with the estimated variance.

Different paths or realizations for the prices are then simulated by drawing random variables from the modelled distribution. The aggregate effect of the simulated realizations can be used to estimate the most probable path or realization for the prices. Additionally, each path may provide clues to the maximum variation in prices that may be observed.

B. Mean reveting models with constant volatility [1]

This type of scheme is commonly referred to as the Vasicek model, where the mean of the process is drawn towards a long term average and the current price is dependent on the interval of time between the current and the former price and a random variable that is typically normally distributed with zero mean and constant variance.

C. Models featuring time varying volatility [1]

These models commonly feature a time dependent tend that may also be mean reverting while also featuring a stochastic volatility component. The volatility may also be modeled via an auto regressive process or may by modeled as an ARCH or GARCH process.

III. PROPOSED PRICING MODELS

The proposed methods more accurately model the random process underlying the movement of the prices by estimating the random variable driving the prices as the first difference of the prices. Since the model and model parameters are estimated using historical data, the estimates must be made on historical time periods that are consistent so that future predictions can be based on past behaviour.

A. Estimating the movement in price

In this model, the random process governing the movement of the prices is modelled as a random walk and is estimated from historical prices. The difference between two prices in the time series then is a sum of random variables drawn from the distribution estimated from the first difference of historical prices. This distribution however can be arbitrary depending on the historical behaviour of the prices and no assumptions are made with respect to its parameters or type.

The random variables are independent and identically distributed (IID) as they are drawn from a single distribution underlying a random walk. The expected value of the change in prices and its variance can then be estimated from the sum of the random variables that govern by how much the prices differ between two samples in the time series of prices. The uncertainty in this measurement can be estimated from the variance. Thus, the variance in the difference between two prices points in the time series is also the variance of a sum of IID random variables. The variance of the sum is also the sum of the individual variances as the random variables are IID.

In this way an estimate for the trend between two price points and a measure of the uncertainty in this estimate can be made.

B. Estimating the random process underlying prices

In this model, the prices are modelled as a random walk where each change to the price is a random variable. The difference between two prices at two points in the time series of prices is then the cumulative effect of a sum of random variables. The random variables governing the movement of the prices is however estimated as the first difference of the time series of prices. The random process governing the movement of prices can be characterized by this distribution of the first differences in historical prices.

Once the random process underlying the prices is so characterized, different paths or realizations of this process can be simulated to assess potential variation in prices and quantify its behaviour. The simulated paths provide insight in to the maximum changes and variance that may manifest in the prices that is consistent with observed historical behaviour. While a diverse array of paths and behaviour may be observed they are all consistent with the estimated random process.

C. Modeling consecutive prices via linear regression

In this contribution we model consecutive prices via a linear regression where the independent and dependent variables are consecutive prices. As movements in prices between successive points in the time series do not change appreciably due to the interaction of a large number of market participants contributing to liquidity, successive prices in a time series do not change appreciably. Large variations would then appear as anomalies.

Successive prices would fall in the general direction of a straight line that can be modeled via linear regression where the parameters of the line are estimated by least squares. In this modeling the time dimension or sequence order of the prices is lost as we consider successive prices in the time series.

The estimated regression line becomes the model for estimating consecutive prices where the next price could be estimated given the current price. The distribution of the error in this estimate can be calculated from the difference between each price point and the line. This distribution provides an estimate of the error in predicting a particular price given the current price level.

D. Impact of measurement technique on measured variance

An accurate estimate of the variance in the prices is important to better estimate risk. It is typically estimated over a window defined as an interval of time. The number of samples observed over such an interval of time is however highly variable due to the nature of the arrival process governing how orders are placed by market participants. Through our analysis we have observed that the variance in price is impacted by the number of trades or samples in the estimate. This time dependence of the estimated variance will result in a less consistent estimate of the variance when the measurements are made over windows defined as intervals of time.

The price can be modeled as a type of random walk particularly when the random variable modeling the change in price is estimated accurately. In such a process the variance depends on the number of steps in the walk or the number of trades or samples in the window of measurement. The measured price is thus expected to have wider variance when longer windows featuring more trades or samples is used.

We thus propose that the variance be estimated over a fixed number of samples or a fixed number of consecutive prices for a more consistent and accurate estimate. The window of measurement may take different forms and may include overlapping as well as non-overlapping intervals. Overlapping windows with significant overlap although more computationally expensive would provide a more stable and better estimate of the variance and volatility.

IV. THEORETICAL UNDERPININGS OF THE MODELS DEVELOPED

In this section we derive the models and the relationships between variables that will be used to generate and validate the results discussed in section VI.

These results and analysis provide the means by which to estimate optimum values for the alert thresholds in the surveillance system in addition to a more consistent measure of the volatility and risks associated with rare events.

A. Estimating the movement in price

The movement in price P, can be modelled as type of random walk expressed by equation (1) where X is an IID random variable drawn from a distribution that can be estimated from the first differences in the prices.

$$P_{n+1} = P_n + X_{n+1} \tag{1}$$

$$P_{n+2} = P_{n+1} + X_{n+2} = P_n + X_{n+1} + X_{n+2} \tag{2}$$

$$P_{n+N} - P_n = \sum_{i=1}^{N} X_{n+i} \tag{3}$$

By this relationship we may estimate the expected value of a difference in prices, N prices apart in the time series of prices and the uncertainty in this measurement. This is accomplished by relating the number of price movements between the two prices to the expected trend between the price points and a measure of the uncertainty around this trend.

The random variable governing the movement in the prices and its distribution F, can be thus estimated from the distribution (5) of the first difference (4) in prices.

$$P_n - P_{n-1} = X_n \tag{4}$$

$$X_n \sim F(\mu_x, \sigma_x) \tag{5}$$

This is a more accurate representation of the random process than which could be obtained by directly measuring the variance in price and associating it with a random variable from a known distribution.

This distribution F may be of arbitrary type with a particular mean and standard deviation corresponding to the random process underlying the prices estimated over the interval of measurement.

The expected value of the difference between two consecutive prices and its variance is given by equations (6) and (7) respectively.

$$E(P_n - P_{n-1}) = E(X_n) = \mu_x \tag{6}$$

$$Var(P_n - P_{n-1}) = Var(X_n) = \sigma_x^2 \tag{7}$$

The expected value and the variance of the change in prices between two point's N prices apart in the time series of prices is given by equations (8) and (9) respectively.

$$E(P_{n+N} - P_n) = \sum_{i=1}^{N} E(X_{n+i}) = N\mu_x \tag{8}$$

$$Var(P_{n+N} - P_n) = \sum_{i=1}^{N} Var(X_{n+i}) = N\sigma_x^2 \tag{9}$$

We may also use this model to better understand the maximum variation that may be expected in the prices, a key quantity in modelling volatility. Insights in to estimating extreme variations between two samples in the time series by relating the mean and variance of the estimate to the number of samples between them can be more reliably obtained via this technique.

The "N" number of samples or prices between the beginning and end prices in the sequence control the degree of uncertainty in the estimation of the final price. Estimation over longer time horizons naturally incur a greater degree of uncertainty and larger error.

B. Estimating the random process underlying prices

The random process underlying the movement in prices modeled as a random walk can be estimated from the distribution of the first differences in the time series of prices.

$$P_n - P_{n-1} = X_n \qquad (10)$$

It is also usual to assume that the first difference in prices is stationary making the distribution underlying random variable X also stationery and invariant to a shift in time. This assumption is in line with modeling X as Normal random variable with a particular mean and variance. Thus, it also commonly assumed that the distribution of X is symmetric with zero mean and constant variance.

In practice however this distribution may not be stationary and be of arbitrary shape and the statistics may vary over time. As such, different distributions can be estimated, each over a stationary period to more accurately capture any non-stationary behavior that may manifest when periods with different behavior overlap.

The distributions estimated over different time periods may be compared via measures such as the Kullback-Leibeler divergence measure [2] to determine the degree to which they differ.

Different paths or realizations of the random process can be simulated by drawing random variables from the estimated distribution. Each realization provides insights into the behavior of the prices, and the maximum variations that may be observed.

The simulation can be carried out by inverting the cumulative distribution function of the distribution and using a random variable drawn from a uniform distribution to reference the corresponding random variable to be drawn in the simulation.

Each path can be used to estimate the potential behavior in the prices and establish limits on its movement that can be employed to optimize and conFig. market surveillance alert thresholds.

C. Modeling consecutive prices via linear regression

Due to the liquidity that arises from the interaction of many market participants, consecutive samples in a time series of prices may not differ much. Hence a linear relationship can most often be established between consecutive samples of the time series of prices. This linear relationship can be estimated by linear regression using the methods of least squares.

Fig. (1) depicts a representation of a time series of prices observed over a time interval $\Delta\ t$ featuring a maximum and minimum. In Fig. (2) however adjacent samples of the time series are considered without consideration to their temporal or sequence order. In this view the sequence order of the series is lost and consecutive samples when plotted against each other appear in the general pattern of a straight line.

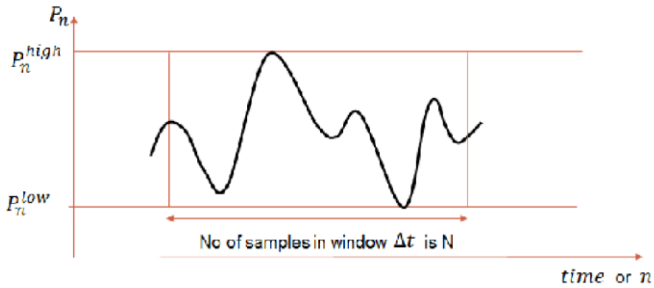

Fig. 1. N sample price segment with unique minimum and maximum

The line of best fit that captures the general pattern in the prices between the minimum and maximum level can be estimated via this modeling. We establish a linear relationship (11) between the current price modeled as the independent variable and a subsequent price as the dependent variable, where "a" and "b" are constants to be estimated and ϵ_n a random variable.

$$P_n = a + bP \qquad (11)$$

the subsequent price $\widehat{P_{n+1}}$ is estimated or predicted as a linear function of the current price as depicted in equation (12).

$$\widehat{P_{n+1}} = \mathrm{a} + bP_n \qquad (12)$$

The parameters (a, b) of the model are estimated by minimizing the sum of the squares of the error ϵ_n between the actual and predicted price.

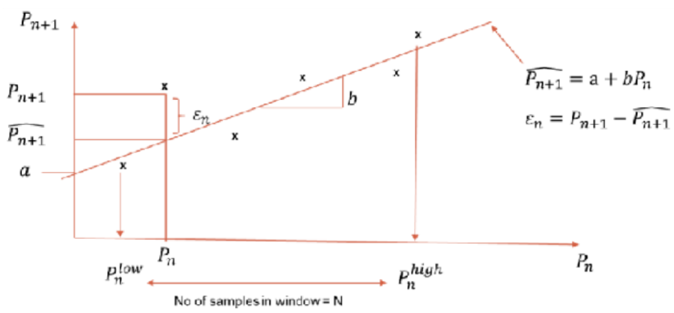

Fig. 2. Linear trend estimated on consecutive prices

The model provides an estimate of the error in estimating each subsequent price given the current price level. Additionally, a trend between the minimum and maximum prices observed over a time interval Δt can also be estimated.

The trend line provides an estimate of the usual change in price that might be expected between consecutive prices in the time series. In this view the change in price between subsequent samples may yet be significant and the usual magnitude of this change captured in the regression line and reflected in its slope.

Thus, the estimated trend line also allows outlier detection with respect to it, where unusual movements in prices could readily be observed. Additionally, the model is sufficiently general to model prices over intervals that are placed more than a sample apart. In this way we may observe the behavior of future prices as a function of the current price level.

This modeling in effect provides an estimate of the conditional probability of observing a future price given the present in way that is consistent with the behavior observed in the past.

D. Impact of measurement technique on measured variance

The variance in the measured price is influenced by the number of executions in the interval of measurement as the process behaves as a random walk.

Since a random walk can be characterized by a sum of IID random variables, the variance too will increase with the number of random variables in the sum. The observed prices are also thus inherently statistically non-stationary.

V. MAIN CONTRIBUTIONS OF THE PAPER

A more accurate estimate of the random process underlying the movement in the prices by estimating the distribution of the first difference in historical prices and estimating price movements as a sum of random variables drawn from this distribution. Distributions of this first difference calculated over intervals of time where statistics are stationary provide a means to better characterize the behavior of the price and also detect when conditions change.

A method to estimate the expected value of the difference between two prices and the uncertainty in this estimate as a sum of random variables, where each such random variable is measured over a segment of historical prices that reflects the behavior over which the estimate is to be made.

A linear model on consecutive prices to estimate a subsequent price point given current price level and error bounds around this estimate.

The ability to linearly relate minimum and maximum prices observed over an interval and estimate the distribution of the error around this estimate.

A more consistent estimate of the variance in prices by making the estimate over a fixed number of consecutive prices in the time series, reducing the dependence of the measured variance on the number of samples in the interval of measurement.

VI. RESULTS

Results that validate each of the models developed in earlier sections are presented to demonstrate the insights that each technique provides.

Additionally, the sensitivity of the measured variance in the price to the number of prices in the measurement is also discussed.

A. Estimating price movements

The difference between two prices spaced N samples apart in a time series may be estimated as a multiple of the expected value of the first differences calculated over a moving window of a certain length. The predicted price N samples from the current price is then N times this average difference added to the current price. This addition of an average of the first differences results from the observation that the first differences of the prices is expected to be stationary particularly in the case of a random walk that is estimated over a representative period.

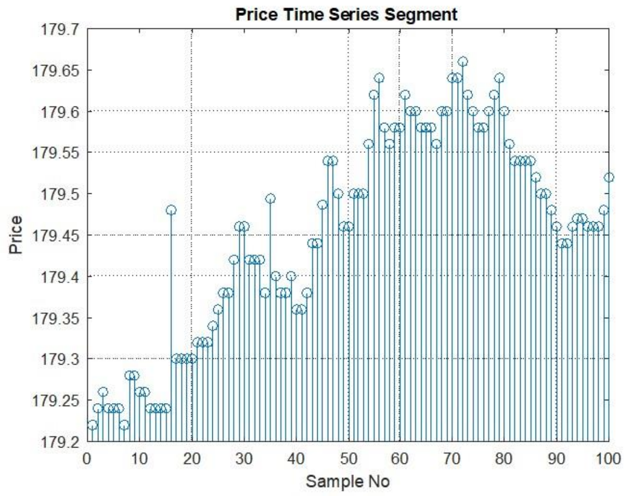

Fig. 3. The segment of a price time series

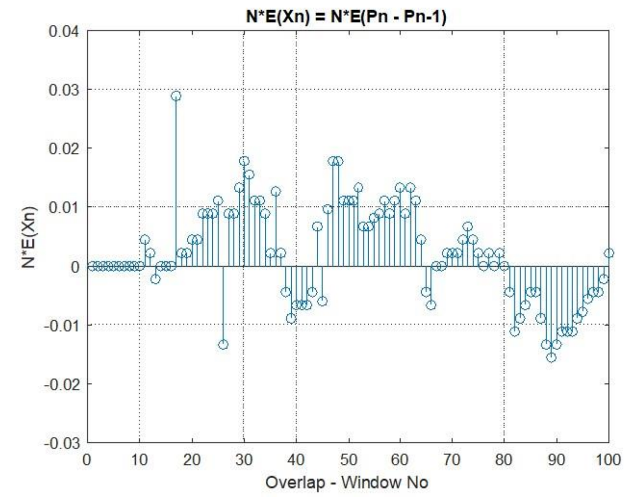

Fig. 4. Expected difference between consecutive (N = 1) prices

The average value of a sequence of the first difference of the prices calculated over a window 10 samples in length is depicted in Fig. (4). Thus sample 11 is the expected value of the first differences in the preceding 10 prices. This average value can then be used to estimate the price of the 11th sample given the 10th sample.

The uncertainty in this estimate is the variance between the current price and the predicted price. An expected value for this variance can be determined as the multiple of (N = 1) and the average variance in the first differences of the prices calculated over a suitable interval such as one the immediately preceding the current price. This quantity is depicted in Fig. 5.

Thus the variance between the current and predicted price increases linearly with N, the number of samples between the two prices. This is symptomatic of the risk or uncertainty in estimating a future price based on historical data.

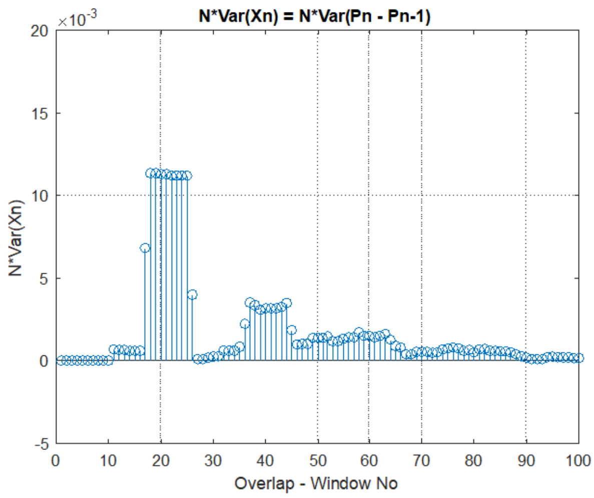

Fig. 5. Expected variance between consecutive (N = 1) prices

Shorter averaging periods or smaller windows over which the random variables are estimated will make the expected first difference and its variance more sensitive to the behavior of the current price. In light of the sensitivity of estimating a future price based on the expectation of the first differences, this expectation should be computed over a sample that is representative of the prices over which the estimate is made.

In fact, the further the samples in the averaging window are from the current price the less relevant they could be towards the estimated price. As such a weighted average that gives more weight to recent prices may also be considered in this regard.

The expected difference in two prices N samples apart is also an estimate of the maximum change that may be expected. In another interpretation, it is also a trend between the two prices.

B. Simulating the random process underlying prices A short segment of the time series of prices depicted in Fig. (6) obtained from liquid security is used to estimate the random process underlying the movement of prices using the technique illustrated in section IV B.

The estimated process is then used to generate new realizations of the price where each simulated path has the same underlying random process as the original price segment. The individual paths may, however, however exhibit widely different behaviors with respect to price due to the nature of the random process governing its behavior.

The random process governing the movement of the price segment and the random process underlying each simulated path is separately estimated and compared in Fig. (8). It is clear that the simulated paths exhibit the same behavior with respect to its random process as the original price segment that was used to model it.

Table 1 provides a summarized view of the original price and the simulated paths. The minimum, maximum, the difference between the maximum and minimum, the mean and variance of each time series is tabulated.

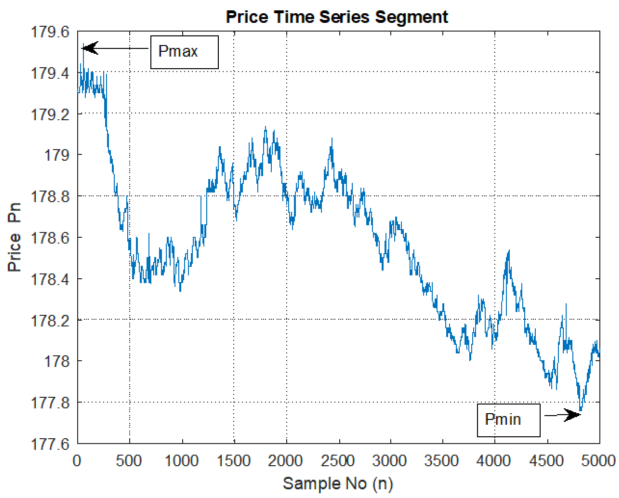

Fig. 6. Segment of price time series

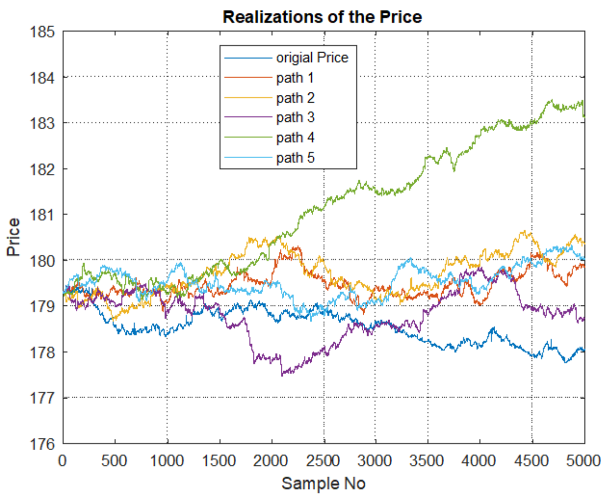

Fig. 7. Paths simulated from original random process underlying price

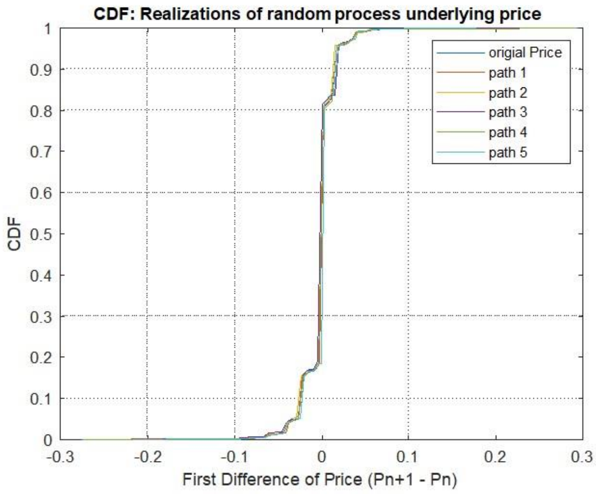

Fig. 8. Cumulative distributions of first differences in price

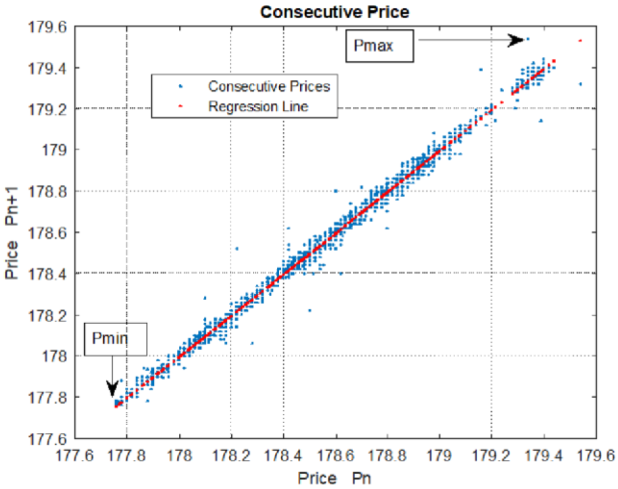

Fig. 9. Consecutive prices and trend estimated via regression

TABLE I. SUMMARY STATISTICS OF SIMULATED PATHS

Statistic	Original	path 1	path 2	path 3	path 3	path 4
Min	179.54	180.31	180.65	179.85	183.51	180.33
Max	177.76	178.82	178.68	177.48	179.14	178.67
max - min	1.78	1.49	1.97	2.37	4.37	1.66
mean	178.55	179.47	179.71	178.82	181.10	179.53
variance	0.14	0.08	0.23	0.31	1.94	0.12

C. Modeling consecutive prices via linear regression

In this section we model a segment of the time series of prices presented in Fig. (6) without reference to its time dimension as discussed in section IV C. This modeling is made possible as the variation between consecutive samples of the price is usually not that significant. This result is used to model consecutive prices as a linear regression particularly in the case of securities exhibiting high liquidity.

Large movements in consecutive prices relative to the usual behavior that has been historically observed may indicate an anomaly or an unusual change in the price. In fact, this model compares each change in price with the changes that have been observed historically as depicted in Fig. (9).

In that respect, the anomaly is observed not with respect to the absolute value of the magnitude of the change but with respect to the magnitude of the change that is usually observed or has been observed historically. This average behavior in the change in prices is captured in the trend line estimated via the linear regression. Thus the subsequent price which modeled as a multiple or a factor of the current price is given by the gradient in the regression line which determines its slope.

Fig. (10) captures the distribution in the error of estimating the subsequent price as a function of the current price, in a way that is consistent with historical price movements. This error is measured as the difference between the regression line and each price. The error, in this case, in this case, is tightly contained around the mean with the low spread as the movement in successive prices is also tightly constrained across the samples used to model this particular price segment. A few outliers are also visible.

The error in prediction also gives insight to how well the regression line captures the general behavior in the trend or average behavior in price movements between consecutive prices. Large errors indicate the presence of unusual movements in price compared to historical movements

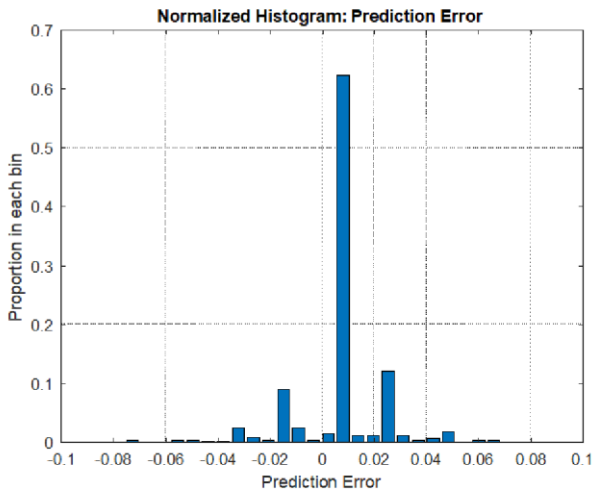

Fig. 10. Distribution of the difference between sample and trend line

D. Impact of measurement technique on measured variance

The following results capture the sensitivity in the measured variance of the price across different techniques of Sensitivity in the number of events to the time interval of measurement Figs. (12) and (13) contrast the impact on the measured variance between using an overlapping window defined as an interval of time and an overlapping window defined as a fixed number of consecutive events respectively.

Measurement windows defined over intervals of time feature a highly variable number of events in each interval of measurement due to the burst nature inherent in the trading process. This burst behavior results in a non-uniform number of events observed over windows that are defined as a fixed interval of time. The mean and the variation in the number of events in a time interval also increases with the length in the time interval over which measurement is made as depicted by the rightward shifting and flattening curves depicted in Fig. (11).

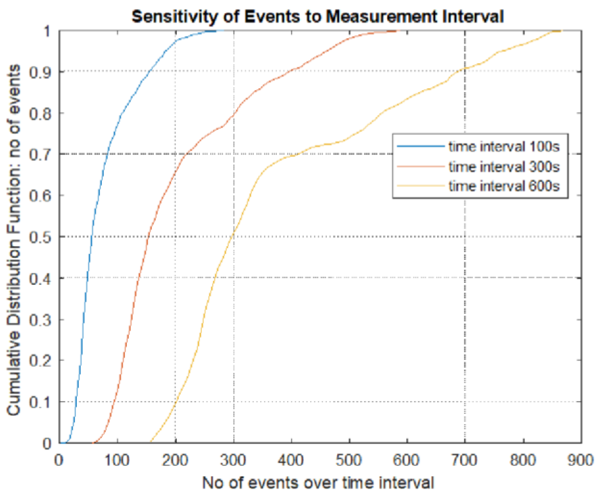

Fig. 11. Sensitivity in the number of events to time interval of measurement

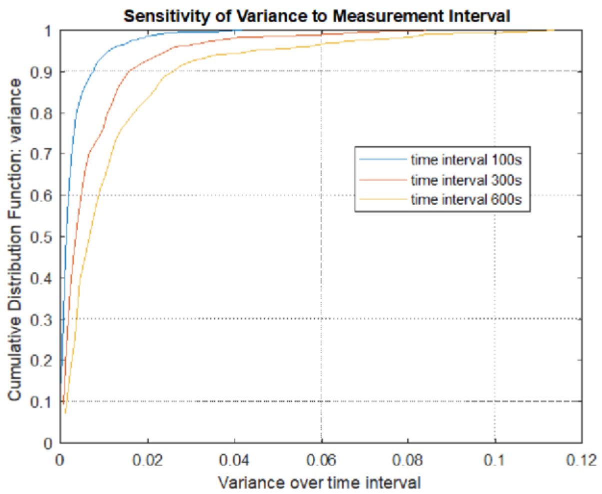

Fig. 12. The sSensitivity of variance to the length of the time interval of measurement

The variance of the price estimated over each window defined as a fixed interval of time increases with the length of the window. This is due to the increase in the average number of events observed in a window with increasing window length. The variation in the variance also increases with the length of the time window as the variation in the number of events in the window also increase. This impact is seen in the flattening of the curves depicted in Fig. (12) with increasing window length.

As discussed in section IV D the variance in the price is influenced by the number of executions due to the random process underlying the movement in prices which makes the prices behave as a random walk that is inherently statistically non-stationary. As a random walk can be characterized as a sum of IID random variables, the variance too will increase with the random variables in the sum.

Overlapping measurement intervals defined as a fixed number of consecutive events on the other hand feature less variation in the variance over different window lengths as the estimate is more consistent, being made over a similar number of samples. As a result the spread in the distribution is narrower and the curves less flat as depicted in Fig. (13)

Thus the variance measurement is more consistent and has lower spread when made over an interval featuring a fixed number of events rather than when made over an interval of time. The dependence of the number of events on the measured variance is also evident in this Fig.

.

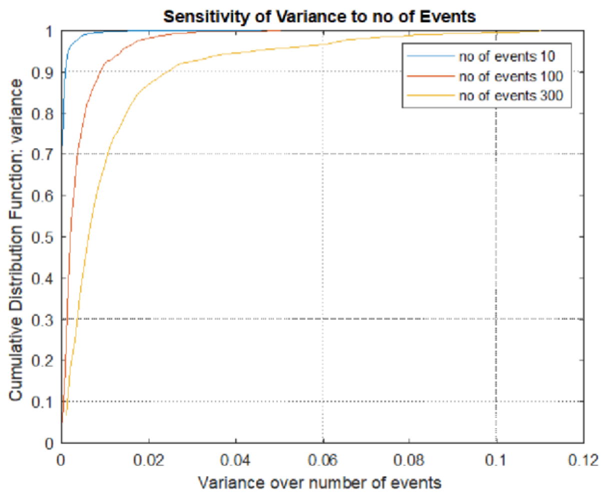

Fig. 13. Sensitivity of variance to number of consecutive events in measurement

VII. APPLICATIONS TO MARKET SURVEILLANCE

Detecting unusual behaviors is a key objective of market surveillance. Thus, unusual movements in price are closely monitored via a system of threshold based alert systems. These thresholds must be set to optimal values to detect anomalies accurately. The models discussed in this paper provide the means to predict potential movement and variations in prices according to historically observed patterns and processes. Thus, they provide a means by which to select optimum values for the alert parameters via a variety of methods. In many detection systems, unusually large movements in the prices occurring over predefined intervals are detected. The interval of measurement over which this change is measured can be defined over an interval of time or as an interval of consecutive prices.

In the first method, the estimation of the expected change in prices a number of samples apart in a sequence of prices and the uncertainty in this estimate allows thresholds to be set to detect this expected change.

In the second method, a historically observed random process is used to simulate new realizations of the prices and each simulated path used to gauge the likely behavior that may manifest. The variations observed in the simulated paths can be employed to set values for the thresholds in a way that is consistent with historically observed behavior.

In a third technique, potential changes in price may also be estimated by modeling consecutive prices via linear regression. In the same way, potential movements that may be observed over a certain number of consecutive events can be estimated by utilizing the regression line to estimate subsequent prices given the current price level. As such limits can also be established on the movement of the price based on historical trends.

VIII. CONCLUSIONS

Three models that provide new insights into the behavior of the prices and its movement were developed in this paper. In one technique the random process governing the moment of the price was estimated and used to estimate future prices given the current price level and uncertainty in this estimate by relating the expected change in price to the number movements between the current and predicted price level. Another model provides an estimate of the future price given the current price and an estimate for the uncertainty and likely error in such an estimate by constructing a regression model between the prices. The model is flexible with respect to the number of price movements between the current and predicted price level and need not be limited to modeling consecutive prices.

Anomalous movements in price may also be more readily detected as prices and price movements can be better compared in relation to historical prices and the trend in historical price movements. A relationship between the minimum and maximum price levels observed over a period of time and a trend between these limits may also be estimated via these techniques.

Realizations of the price simulated by estimating the random process underlying historical prices provide yet another way to estimate unusual variations in price that may manifest in the future in a way that is consistent with the observed historical behavior. This modeling also provides a better estimate of volatility that leads to a more accurate estimate of the risks associated with financial instruments and portfolios constructed using volatile securities, leading to more accurate pricing. Thus, these models also enable more effective market surveillance by estimating optimum parameters for software alert systems that monitor unusual price movements while giving market participants a way to benefit from more accurate models of price, price movements and risk.

REFERENCES

[1] John C. Hull, “Options futures and other derivatives,” Perarson, Prentice Hall, NJ., 2006, pp 265-280, 461-483.

[2] https://en.wikipedia.org/wiki/Kullback%E2%80%93Leibler_divergence Kulback-Leibler divergence measure, extracted on 10 June 2019.

[3] Martin Baxter and Andrew Renine, “Financial Calculus,” Cambridge University Press, Cambridge., 2005, pp 44-81.

[4] Jamil Baz and Geooge Chacko, “Financial Derivatives,” Cambridge University Press, Cambridge., 2004, pp 5-59.

[5] Yi Cao, Yuhua Li, Sonya Coleman et al. “Adaptive hidden markov model with anomaly detection for price manipulation detection”, IEEE transactions on neural networks and learning systems, 2014. [6] Yi Cao, Yuhua Li, Sonya Coleman Ammar Belatreche, “Detecting price manipulation in the financial market”, University of Ulster, Londonderry, United Kingdom.

[7] Teema Leangarun, Poj Tangamchit & Suttipong Thajchayapong, “Stock price manipulation detection based on mathematical models”, International journal of trade, economics and finance, vol 7, no 3, June 2016.