A Framework for Dynamic Pricing Electricity Consumption Patterns via Time Series Clustering of Consumer Demand

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Abstract— **A time series is a quantity sampled regularly in time where the techniques used in its modeling and characterization are dependent on the complexity of its statistical properties. Domestic electricity consumption exhibits time series in the form of time varying usage patterns. These patterns are of fundamental importance in determining load based pricing schemes and in more effective generation and distribution planning.**

Clustering individual household electricity consumption patterns enables a utility to design pricing plans catered to groups of households in a particular locality to more accurately reflect the cost of supply at a particular time of day. Dynamic pricing is an attempt to change the consumption behavior to one that is more uniform and devoid of sharp peaks ensuring a more uniform utilization of generation capability avoiding idle capacity at times of low demand.

In light of this we model the movement of collections of such time series in order to observe the relative movements between them with a view to understanding their group behavior and characteristics not otherwise outwardly visible.

Time series clustering can be accomplished by directly clustering samples of the series using an established method of clustering or one could estimate various representative characteristics of each series that are then used to parameterize the series and form the attributes used in one of the standard clustering techniques.

In this paper we model each time series as an Autoregressive Moving Average (ARMA) process with an optimal model order determined by the Akaike Information Criterion when the parameters estimated by the Hannan-Rissanen algorithm converge. The estimated model has the representation of a transfer function with a frequency response defined by the ARMA parameters. We use the frequency response as the means to further refine the within cluster profiling and classification of the objects.

Through our modeling we are also able to identify instances where the consumption behavior exhibits patterns that are uncharacteristic or not in line with the behavior or consumption profiles of the other households in a particular locality providing insights in to potential faults, fraud or illegal activity.

Keywords— Time Series, ARMA, Clustering, Stochastic Processes

I. INTRODUCTION

Electric power utilities are faced with the challenge of delivering an uninterrupted power supply in the face of widely varying demand. This variation in demand which exhibits broad seasonal patterns across the day, day of week and longer periods also depend on working hours, holidays and climatic conditions. As many consumers consisting of both households and businesses react to these conditions at the same time they create a seasonal pattern in the demand for electricity. This seasonal demand for electricity exhibits periods of very high demand or peaks and low demand at other times. The utility then has the challenge of meeting the demand for peak power at different times of the day and so has to dimension the generation and distribution capacity to meet this seasonal peak in demand. A more uniform distribution in demand would on the other hand enable the utility to have lower generation and distribution cost as the required peak generation capability is lower requiring comparably lower investment.

In the Sri Lankan context, utilities often rely on thermal power to augment the hydro power output in times of peak demand especially when the demands on the reservoirs are high or the need to conserve water is great. This reliance is an additional burden on the country due to the relatively high cost of fuel oil and coal. Thus the ability to regulate demand to some extent so that the total demand on the utility at its peak hour is considerably reduced will greatly help in reducing the overall cost to the utility and the country in general.

In light of this, smart meters are being introduced to remotely measure the demand for electricity and the power consumption patterns of the consumer at intervals of up to 15 minutes. These measurements provide the utility with a near real time view of consumer demand and a good understanding of historical patterns in demand.

As with any product which is subject to market forces the laws of supply and demand impact and determine its price. Thus the cost of production and supply of electricity is higher at times of peak demand than when the draw on the network is low. The goal of the utility is then to price the supply of electricity at a cost which is reflected in the cost of production. This leads to a variable pricing scheme for the supply of a Kilowatt of power depending on the variation in demand that typically exhibits a seasonal pattern. In other words the cost of production will vary with time of day and therefore the price of supply too should reflect this variation.

Pricing electricity to reflect the cost of production will help to reduce the seasonal peak in the demand to some extent as consumers shift their consumption pattern to take advantage of lower costs at different times of the day. The shift in consumption pattern to a more uniform one will depend on the degree of incentive offered to consumers by appropriately pricing the supply at different times of the day.

The advent of smart equipment and network controlled household equipment and devices will help consumers to use these incentives to better plan how they operate their electric devices. The internet of things (IOT) makes remote monitoring and operation of electric devices easier to schedule according to a pre-arranged time table that takes advantage of lower electricity prices.

These initiatives require the utility to understand the consumption patterns in different types of households, businesses and other organizations. The demand which takes a time varying pattern is best understood and modeled as a time series. Collections of these time series will exhibit broadly similar behaviors as they represent consumers with similar circumstances giving rise to similar consumption patterns. Grouping consumers based on their consumption pattern is the first step towards creating consumer specific billing plans or tariff structures catering to a particular segment of the market.

In this paper we propose a novel procedure for grouping collections of time series with the aim of identifying those that exhibit similar variations or patterns in time. Specific incentives can then be designed for each type of group depending on their particular consumption pattern. These incentives also have the objective of reducing the peak in the consumption pattern of the consumers and creating a more uniform power utilization profile.

II. EXISTING TECHNIQUES FOR CLUSTERING TIME SERIES [5]

Time series may be grouped according to the similarity of each sample in one series with the corresponding sample in the other series using traditional cluster analysis methods, or in a second approach features may be estimated for each series and the resulting features clustered or in a third approach a model representative of the time series estimated and the models used as the means for comparison and grouping the time series.

A. Direct Clustering

Traditional cluster analysis algorithms like K-Means, Fuzzy C means and Hierarchical clustering can be employed to form groups of time series by making comparisons between the corresponding time samples of each series. In these approaches each time series is compared with the rest using a measure of “similarity” or “distance” to determine how “close” or “similar” each series is to the rest. In these methods however each sample is considered a feature and depending on the length of the series there could be hundreds if not thousands of features.

This approach therefore is not ideal for this type of problem as a time series is more akin to a sampled version of a single feature and not a collection of features where each feature corresponds to a sample in time.

B. Feature Estimation & Clustering

Each time series could be summarized by estimating quantities representative of each series like its minimum, maximum, mean and other descriptive statistics. This is in effect a process of dimensionality reduction where we represent each series with a collection of representative features where the number of features is considerably less than the length or number of samples in the time series.

These features can then be clustered using one of the traditional clustering algorithms to create groups of time series that behave in similar fashion based on the features selected to profile each time series.

This method too is not ideal in that the entirety of the time series is not modeled but a feature extracted from the time series used in its characterization. Thus certain nuances in the behavior of the time series will not be captured and in any event the features cannot capture the holistic behavior of the time series giving rise to a rather rudimentary representation of their group behavior.

C. Model based Clustering

In this approach a model that can represent the overall behavior of the time series is created. The models then become the means by which the comparison between time series is made. Models can be compared with respect to their complexity or model order, model parameters and other characteristics that arise as a result of the way the model is configured.

The random process underlying a time series captures both its time and frequency behavior. Modeling the random process is then a means by which time series can be compared. Autoregressive (AR), moving average (MA), autoregressive integrated moving average (ARIMA) are some of the models that can be used to estimate the random process underlying a time series [1].

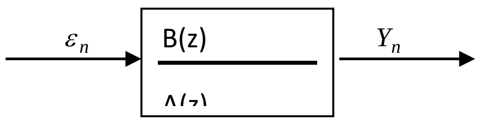

Fig.1: System function representation of an ARMA process

In these approaches the time series Yn is modeled as being the result of a filtered sequence of independent identically distributed (IID) random variables (ε_n also called a white noise source, most often assumed to be drawn from a normal distribution) together with delayed versions of the time series Yn itself. These models are a powerful yet compact way of capturing the dynamics underlying complex behavior both in the time and frequency domains.

III. A NOVEL TIME SERIES CLUSTERING ALGORITHM

We propose an algorithm that models the generative process underlying the time series of the selected features. In our modeling we treat each time series as being the result of an autoregressive moving average (ARMA) process. Such processes are ideally suited to model complex patterns and large variations in the observed time series in models of low complexity. In this regard the estimated model is robust with respect to the length and the complexity in the exhibited pattern.

The algorithm estimates a series of ARMA models with increasing levels of complexity for each time series using the Hannen-Rissanen algorithm to estimate the model parameters in each case. Models with different levels of complexity provide different error sequence behaviors and convergence performance for the estimated ARMA model parameters. The proposed algorithm selects the most parsimonious model for which the model parameters converge to a steady state.

In models of this type the error convergence performance improves with increasing model complexity. More complex the model the tighter the bound on the error sequence. This tradeoff between model complexity and variance in the error sequence is captured in the Akaike Information Criterion (AIC) measure.

The AIC is used to determine the simplest ARMA model for which the error sequence is bounded to within a specified tolerance. In other words we select the model with the lowest AIC value for which the model parameters converge. The convergence of the ARMA model parameters is established when the ratio of the variance to the mean of each parameter is bounded to within 10%.

A. The Hannen-Rissannen Algorithm [3]

When the underlying process is locally stationary, the parameters a_i and b_j given by equation (1) are constant. When the process is non-stationary the parameters will be time varying. In such a case where the source is non-stationary a sample by sample estimate of the parameters can be made. Using a model described by a set of constant parameters to model a statistically non stationary process trades the amount of error tolerable with the degree of non-stationarity in the modeled data source. An ARMA(p,q) process can be described via equation (1)

$$Y_n = a_1 Y_{n-1} + \ldots + a_p Y_{n-p} + \varepsilon_n + b_1 \varepsilon_{n-1} + \ldots + b_q \varepsilon_{n-q} \tag{1}$$

where ϵ_n is a sequence of zero mean serially uncorrelated random variables with finite variance (white noise) uncorrelated with Y. In some incarnations we may insist on the noise sequence to be independent identically distributed (IID) as well.

Taking Z transforms (alternatively one can use the backward operator "B" as is found in the time series literature)

$$[1 - a_1 z^{-1} \ldots - a_p z^{-p}] Y(z) = [1 + b_1 z^{-1} \ldots + b_q z^{-q}] E(z) \tag{2}$$

We arrive at a "system transfer function"

$$\mathrm{Y(z)} = \frac{[1 + \mathrm{b}_1 z^{-1} + \ldots + b_q z^{-q}]}{[1 - \mathrm{a}_1 z^{-1} - \ldots - a_p z^{-p}]} E(z) \tag{3}$$

Rewriting (2) & (3) as a high m[th] order pure AR process given by equation (5)

$$\theta(z) Y(z) = E(z) \tag{4}$$

$$Y_n = \theta_1 Y_{n-1} + \ldots + \theta_m Y_{n-p} + \varepsilon_n \tag{5}$$

$$\theta(z) = 1 - \theta_1 z^{-1} - \theta_2 z^{-2} - \ldots - \theta_m z^{-m} \tag{6}$$

Formulate the parameter estimation of the high order pure AR process via Yule - Walker equations via equation (7)

$$E\{Y_n Y_{n-h}\} = \theta_1 E\{Y_{n-1} Y_{n-h}\} + \ldots + \theta_m E\{Y_{n-m} Y_{n-h}\} + E\{\varepsilon_n Y_{n-h}\} \tag{7}$$

the autocorrelation function is given by (8) when lag h ~= 0

$$r(h) = \theta_1 r(h-1) + \ldots + \theta_m r(h-m) + E\{\varepsilon_n Y_{n-h}\} \tag{8}$$

when $h = 0$ at zero lag the autocorrelation is given by (9)

$$E\{\varepsilon_n Y_{n-h}\} = \sigma_\varepsilon^2 \tag{9}$$

$$r(0) = \theta_1 r(h-1) + \ldots + \theta_m r(h-m) \tag{10}$$

In the case of real processes $r(h) = r(-h)$

The parameters θ_i can be estimated from the Yule-Walker equations via equation (11)

$$\begin{bmatrix} \mathrm{r}(0) & \ldots & r(m-1) \\ \ldots & \ldots & \ldots \\ r(m-1) & \ldots & r(0) \end{bmatrix} \begin{bmatrix} \theta_1 \\ .. \\ \theta_m \end{bmatrix} = \begin{bmatrix} r(1) \\ \ldots \\ r(m) \end{bmatrix} \tag{11}$$

Once a preliminary estimate for the autoregressive parameters

$\theta(z)$ has been made, the error sequence can be estimated by substituting in equation (5) rearranged as equation (12)

$$Y_n - \theta_1 Y_{n-1} - ... - \theta_m Y_{n-m} = \varepsilon_n \tag{12}$$

and the error sequence at different lags calculated via (12) is depicted in the (13) through (14)

$$Y_{n-1} - \theta_1 Y_{n-2} - ... - \theta_m Y_{n-m-1} = \varepsilon_{n-1} \tag{13}$$

$$Y_{n-p-q-1} - \theta_1 Y_{n-p-q-2} - ... - \theta_m Y_{n-p-q-m-1} = \varepsilon_{n-p-q-1} \tag{14}$$

The estimation of parameters of the original model equation (1) given by

$$Y_n = a_1 Y_{n-1} + ... + a_p Y_{n-p} + \varepsilon_n + b_1 \varepsilon_{n-1} + ... + b_q \varepsilon_{n-q}$$

can then be put in the form of a least squares parameter estimation problem where

$$\begin{bmatrix} y_{n-1} & \cdots & y_{n-p} & \varepsilon_n & \cdots & \varepsilon_{n-q} \\ \\ \\ \\ \\ y_{n-p-q-1} & & & \varepsilon_{n-p-q} & & \varepsilon_{n-p-2q} \end{bmatrix} \begin{bmatrix} a_1 \\ \cdots \\ a_p \\ 1 \\ b_1 \\ \cdots \\ b_q \end{bmatrix} = \begin{bmatrix} y_n \\ \cdots \\ y_{n-p+1} \\ y_{n-p} \\ \cdots \\ \cdots \\ y_{n-p-q} \end{bmatrix} \tag{15}$$

that takes the form

$$M\theta = m \tag{16}$$

$$\theta = (M^T M)^{-1} M^T m \tag{17}$$

θ can then be estimated by iterating until the parameters converge (or in our case we iterate a certain maximum number of times) and determine the Akaike Information Criteria value for that specific ARMA(p,q) model.

B. Akakike Information Criterion

The Akaike information criteria (AIC) provide a means for estimating the model order of a ARMA(p,q) random process. The AIC trades the variation in the error sequence with the number of parameters used to define the model or its degree of complexity.

As the number of parameters increase the variance in the error should decrease, and the AIC measure provides a convenient means for arriving at a suitable model order by trading this decrease in the variance of the error with the increase in the complexity (number of model parameters) of the model.

AIC = log(error variance or sum of squared prediction error) + 2*(p+q) (18)

The number of parameters (p+q) are a penalty term that compensates for the lower bounds on the error sequence that results with increasing model complexity.

C. Model Selection

A series of ARMA(p,q) models of increasing complexity are estimated for each time series via the Hannen-Rissanen algorithm described above and the resulting AIC measured for each model. We select the model that provides the minimum AIC for a set of converged parameters.

The time series are grouped according to the ARMA(p,q) model that best describes each series.

D. Frequency Response[6]

The ARMA model estimated for each time series captures the essence of its variation in time in a relatively low dimensionality model when the complexity of the time series is low. In this interpretation the time series is the result of a delayed version of itself which is driven by a white noise process. A white noise source is in theory one that contains all frequency components but for practical purposes it is typically a band limited signal.

The ARMA model has another interpretation, that of a digital filter with a phase and frequency response determined by the values of its model parameters. The frequency response indicates the degree to which each frequency in the input spectrum is impacted (amplified or attenuated) when it appears at the output. Since these two interpretations give rise to the same model we can conclude that the time series is in effect a filtered version of itself driven by a white noise source where the frequency response of the ARMA model determines how the frequencies are impacted in the output spectrum.

Thus a time series with high frequency components is likely to be the result of an ARMA model with a frequency response that allows high frequencies. A time series that is predominantly low frequency in character is likely to be produced by an ARMA random process that does not attenuate low frequencies, such as low pass filter.

Since the parameters of the model determine its frequency response, and the parameters may differ from series to series the frequency response need not be unique across models of a particular complexity.

IV. ANOMALY (OUTLIER) DETECTION CRITERIA

Another result of this modeling is the ability to detect those power consumption patterns that exhibit behavior significantly different from the rest. Outliers in this context are those time series that may be the result of power consumption entities that are anomalous in the kind of power consumption equipment or feature some internal fault or perhaps engaged in fraud.

We group the time series by ARMA models of a particular complexity. Outliers can be identified as those groups of patterns that are very few in number that fit a given order of ARMA model complexity.

Within a particular class of model complexity we can also compare the ARMA parameters of each time series to determine how similar individual time series are to each other. In this way we may further refine the grouping of the time series within a particular class of model complexity by clustering the ARMA parameters of each series to obtain groups of time series. Through this process we are also able to identify time series that are very similar and those that are very different from the rest of the series within a given class of model complexity.

V. RESULTS

Average daily power consumption patterns of 100 entities sampled every 15 minutes are analyzed to demonstrate the operation of the algorithm. Each time series is modeled independently and those that fall in to a particular class of model complexity are grouped together.

The time series within each class of model are further analyzed to determine those that have parameters that most closely resemble each other. Those time series that belong to a particular class of model and at the same time are described by parameters of similar value are those that exhibit similar power consumption patterns. These series will be similar both in time behavior and frequency content as the parameters of the model also determine its frequency response.

Entities with power consumption patterns that fall in to particular clusters can be offered a common tariff depending on the time of day the peak consumption patterns of the group occur. The elasticity of demand for electricity will be a function of time in addition to its price. Thus the relationship between quantity demanded and the price of a unit of electricity will take different forms depending on the time of day, locality and other demographic and econometric factors. The time based tariff structure designed to alter the consumption pattern of the group reflects these concerns.

A. Total (average) power consumption profile

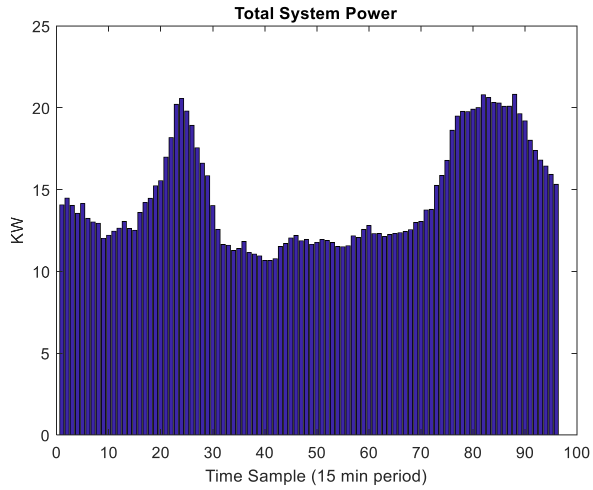

Fig.2: Total power consumtion of all 100 entities

Figure 2 presents the total power consumption pattern of all 100 entities. We notice two clear peaks around 5:00 AM - 7:30 AM (corresponding to time samples 20 and 30) and 7:00 PM – 10:30 PM (corresponding to time samples 76-90).

B. Time series grouped by model complexity

Figure 3 depicts the time series that are best described by an ARMA(1,2) model featuring one autoregressive parameter and three moving average parameters. There are 68 time series that belong to this group of the original 100, each identified by its ID.

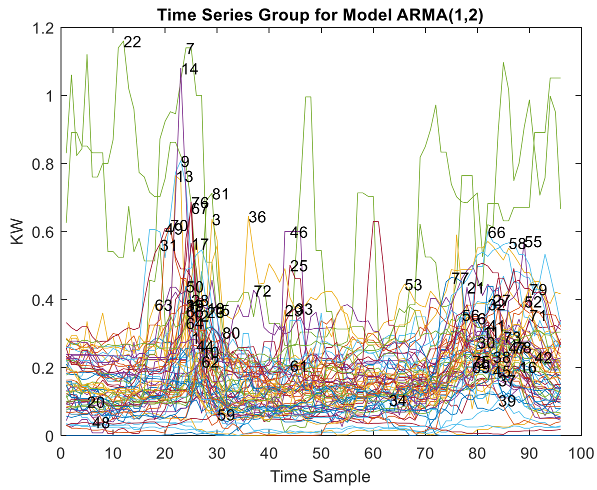

Fig.3: Time series that fit models of complexity ARMA(1,2)

Figure 4 presents the autoregressive and moving average parameters of the collection of time series depicted in Figure 3, each modeled as an ARMA(1,2) processes of the form given by equation 19.

$$Y_n = a_1 Y_{n-1} + \varepsilon_n + b_1 \varepsilon_{n-1} + b_2 \varepsilon_{n-2} \quad (19)$$

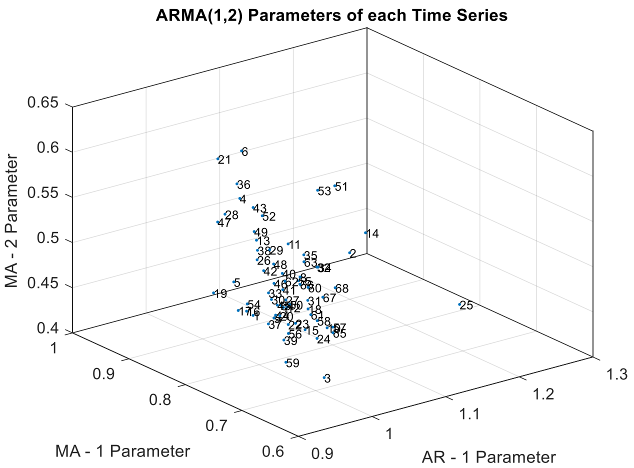

Fig.4: Parameters of time series that fit models of complexity ARMA(1,2)

We observe that most ARMA(1,2) parameters are clustered together as depicted in figure 4 as the time series in the group are broadly similar. Thus most parameters are very similar to each other with the exception of a few time series like number 25 that has parameters significantly different from the rest. We may identify time seies 25 as an outlier from within the broad group of patterns that fall in to the class of models with complexity ARMA(1,2).

We notice that the consumption pattern of this series depicted in figure 5 is different from the average power consumption paterns of the majority of the series with a peak arround time sample 40-48 coresponding to 8:00AM – 10:00AM.

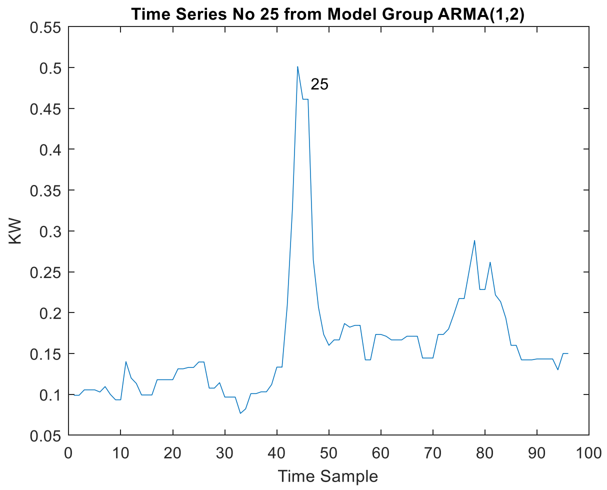

Fig.5: Outlier power consumption pattern No 25 of group ARMA(1,2)

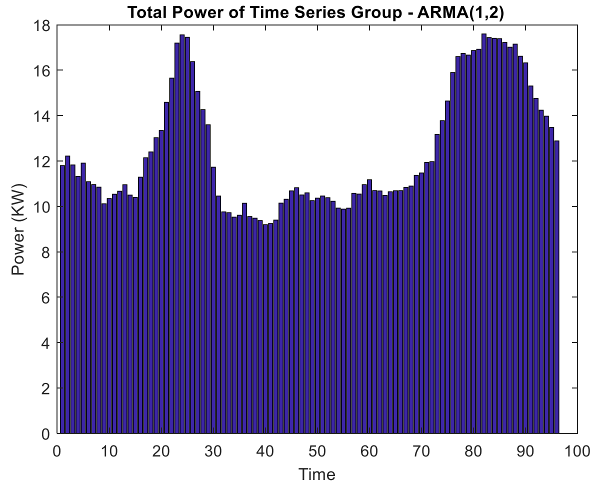

Fig.6: Total power of all time series with model complexity ARMA(1,2)

Figure 6 depicts the total power consumption of all time series modeled as ARMA(1,2) processes and depicted in figure 3. We observe that the pattern in the outlier time series number 25 depicted in figure 5 is different to the average behavior of the ARMA(1,2) group to which it belongs.

Figure 7 depicts the time series that are described by models of complexity ARMA(3,4). Since there are two time series in this group, we may consider them outliers as they are relatively few in number and belong to a class of model that is different from the other models.

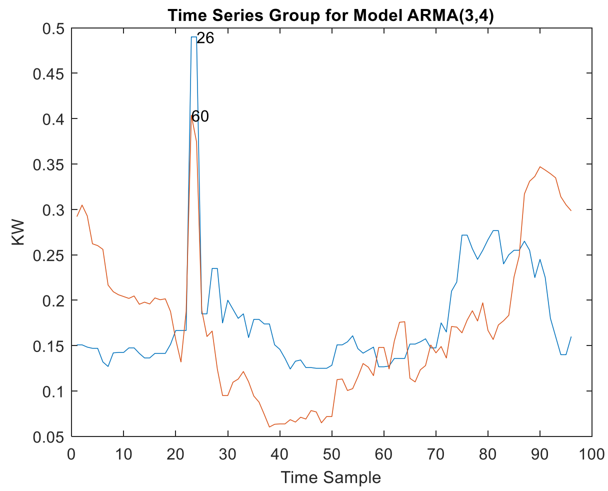

Fig.7: Time series that fit models of complexity ARMA(3,4)

The autoregessive and moving average parameters of the two series numbered 26 and 60 are given in the two arrays below.

[0.8369 -0.7429 1.0646 1 0.8309 0.6752 0.5491 0.3363] &
[0.9325 -0.8261 0.9477 1 0.8137 0.6589 0.5539 0.3765].

The two sets of parameters are reasonably close and the underlying two time series also display a reasonbaly close similarity of pattern.

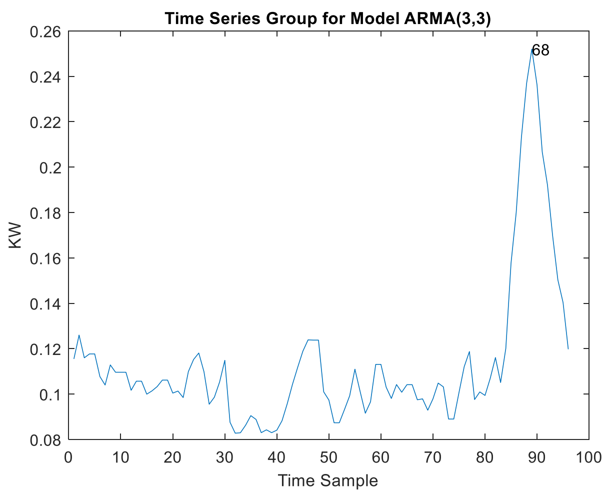

Fig.8: Time series that fit models of complexity ARMA(3,3)

Figure 8 depicts the time series that are descried by models of complexity ARMA(3,3). As there is only a single time series in this group, we may consider it an outlier as well. We also observe that the consumption pattern is clearly different from the other patterns with a single peak arround sample 90, which corresponds to a time 10:30 PM.

C. A time based tariff structure that reflects elasticity of demand to price

There is a negative relationhip between quantity demanded and price as depicted in Figure 9 and we expect the form of the relationship to change with time of day within a particular cluster of power consumpton entites. Thus the rate of change of quantity demanded with price will also vary with time of day. Hence the price discount required to reduce demand will also depend on the time of day.

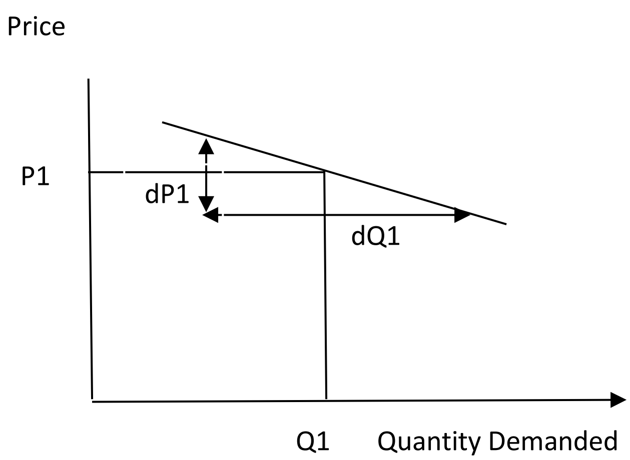

Fig.9: Price sensitivity of electicity demand at a given time (low demand)

In one strategy we may offer a discount to a group of entities in a cluster such that the peak demand is flattened. A discount on a unit of power applied to time periods both before and after the peak can accoplish such an effect. Different discounts will apply for the periods both before and aftter the peak depending on the sensitivity of quantity demanded to a change in price. The discount can also be determined by taking in to account the total reduction in peak energy that is desired and the price elasticity of demand.

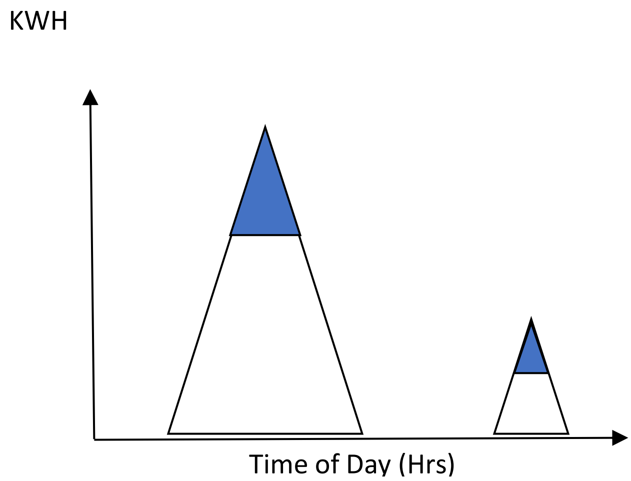

Fig.10: Time varying peak demand of the selected group

Figure 10 depicts a typical peak demand pattern of a group of entities.

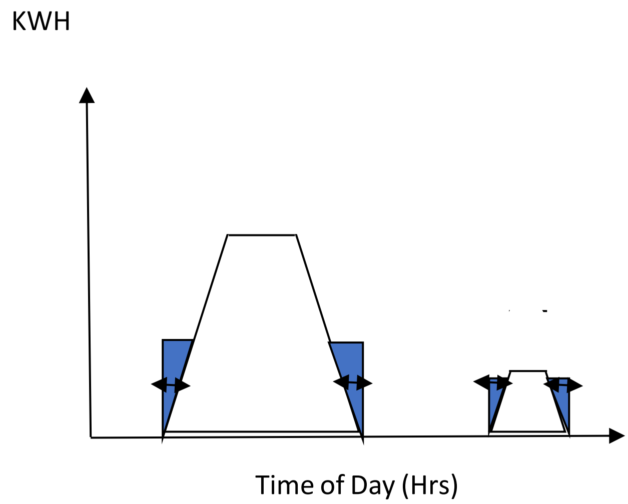

Fig.11. Load balancing by shifting demand for the group depicted in Fig 10

Figure 11 depicts how the peak demand may be lowered by shifting demand to time periods before and after the peak by offering a discount over the periods indicated by the double arrows. For a given discount, higher consumption will be observed in time periods further away from the time at which the peak demand occurs due to a comparatively lower cost. The price per unit will be highest at the peak demand and drop to lower values in periods both before and after the peak. As a result the total quantity consumed will be low at the peak and the amount consumed will rise on either side of the peak.

We would expect a gradually increasing power consumption pattern for time periods after the peak and a gradually decreasing power consumption pattern in time periods approaching peak demand. In this way we may envisage a scenario where the total power consumption remains unchanged while the peak demand is reduced by shifting the demand to periods both before and after the peak resultng in a more uniform overall power consumption pattern as depicted in Figure 11. It is expected that the entites have smart devices that allow scheduling tasks to other times.

Figure 9 illustrates how the price elaticity of demand can be calculated using the relationship beteen quantity demanded vs. price corresponding to a specifc time of day. The curve is one that depicts a slope that is not steep and the relationship is elastic as the percentage change in quantity demanded is larger than the corresponding percentage change in price.

The price elasticity of demand (PED) is the percentage change in quantity demanded as a ratio of the percentage change in price.

$$PED = \frac{dQ1/Q1}{dP1/P1} \tag{20}$$

Assuming that this relationship holds for a period of low demand and that we wish to shift a quantity of demand Q to this period, we can determine the required percentage change in price %P by substituting Q for dQ1 and inverting equation (20) to get equation (21).

$$\%P = dP1/P1 = \frac{Q/Q1}{PED} \tag{21}$$

VI. Conclusion

Through our modeling we establish that the proposed algorithm can group time series according to the broad similarity of pattern in its observed time behavior and frequency content.

The algorithm estimates an autoregressive moving average model of the lowest complexity that best represents the underlying random process governing each time series. The estimated model is thus optimal in the sense of achieving the optimal tradeoff between the bound on the variance of the error sequence and the number of parameters used to define the model.

Each model is thus described by a set of ARMA parameters that also describes the time behavior and frequency content of the modeled time series. Thus time series that exhibit a similar pattern would have a similar set of ARMA parameters within a given class of model complexity. Those time series that are different would as a consequence have ARMA model parameters different from the rest.

Groups or clusters of time series can therefore be created by clustering the parameters of the ARMA models of a particular complexity. Through this process of clustering it would be possible to further refine the grouping at each level of complexity and also help identify outliers [4].

While large groups of time series that exhibit similar behavior with respect to power consumption patterns would enable the utility to create incentives to suit each segment of the market, outliers may provide insights in to unusual power consumption entities or fraudulent activity.

The incentive offered can take the form of a group discount applied to the entities of each cluster taking in to account the total amount of energy redistribution desired and the price elasticity of demand.

References

[1] Z. Brezezniak & T. Zastawniak, Basic Stochastic Processes, 1st ed., vol. 1. London: Springer, 2000, pp.85-137.

[2] F.M. Reza, An introduction to information theory, 1st ed, vol. 1, New York: McGraw-Hill, 1994, pp. 77-130.

[3] Peter J. Brockwell & Richard A Davis, Introduction to Time Series and Forecasting, Srpinger 2000, pp 156-157.

[4] Matteo Mateucci, A tutorial on Clustering Algorithms: K-Means,” extracted on March 3, 2017.

[5] T. Warren Liao, Clustering of time series data – a survey, Elsevier Pattern Recognition Society.

[6] Robert D Strum & Donald E. Kirk, Discrete systems and digital signal processing, Addison Wesley, 1989.

[7] Christian Gourieroux, Financial Econometrics, New age international publshers, 2005